Frequency Domain Computation of Eddy Currents in Superconductors

S. Mezani, B. Douine, T. Lubin, J. Lévêque and A. Rezzoug

GREEN-UHP, Faculté des Sciences et Technologies
BP 239, 54506, Vandoeuvre-lès-Nancy, France
E-mail: smail.mezani@green.uhp-nancy.fr

***Abstract* — The concept of effective resistivity in electrically non-linear media is used to treat problems expressed in terms of phasor quantities under a sinusoidal time varying excitation. 2D and 3D computations of the magnetic field in High Temperature Superconductors (HTS) is carried out using an A-V formulation. Results of ac loss computation in an HTS tube under self field conditions are presented and discussed.**

***Index Terms*—Effective resistivity, finite element methods, frequency domain, High Temperature Superconductors**

I. INTRODUCTION

High Temperature Superconducting (HTS) materials are characterized by highly non-linear electric properties which are generally described by an E-J power law (E is the electric field norm and J the current density norm).

Time-stepping techniques using different formulations can be used [1]-[3]. The authors reported certain difficulties concerning the convergence of the numerical FE schemes.

Another approach to solve diffusion equations for systems driven with a sinusoidal input (currents and voltages) is to use a time-harmonic formulation. Complex representation (phasor quantities) of the fields is used and only one computation is needed to get a steady state solution. Although strictly valid for linear problems, the effects of nonlinearities in frequency domain approaches are included on the "fundamental" of the response so an effective media is introduced.

In problems involving magnetic materials (saturation and hysteresis), an effective magnetization curve which originates from the dc non-linear B-H curve is obtained by some averaging rules [4]. In this paper, the authors investigate the application of the time-harmonic method in 2D and 3D problems that involve non-linear E-J laws. The effective electric resistivity concept is introduced. Results for ac losses computation under self field conditions are reported and discussed.

II. EFFECTIVE RESISTIVITY CONCEPT

We need while using phasor quantities in non linear problems to specify how the complex representations of the fields are dependent. In the general case, the electric field *e(t)* and the current density *j(t)* present time harmonics. Even when dealing with only two harmonics of *e* and *j*, we are led to very complicated calculus.

Hence, one of the two following hypotheses is considered:
- *e* is sinusoidal and *j* is multi harmonic
- *j* is sinusoidal and *e* is multi harmonic

Given a *E-J* characteristic of an HTS material, one can compute the evolution of the harmonics of the electric field for a sinusoidal current density. Reciprocally, one can compute the harmonics of the current density for a sinusoidal electric field. The power law describing an HTS material is:

$$\left(E/E_c\right)=\left(J/J_c\right)^N \tag{1}$$

where E_c is the critical electric field (usually E_c = 10^{-4} V/m) and J_c the critical current density. $N=1$ corresponds to a normal conductor and $N=\infty$ represents the critical state model suggested by Bean.

Independently of the used formulation to solve the diffusion equation in a superconductor, effective *E-J* curves are obtained using different averaging rules [5].
If the electric field *e* is sinusoidal with a pulsation ω, we state that $e(t)=E\,sin(\phi)$. The Fourier series expansion of the current density *j* gives $j(t)=J_1\,sin(\phi)+.......$ with $\phi=\omega t$.

A first definition of the effective *E-J* curve is the one which links J_1 to *E*. The first harmonic of *j* being computed by:

$$J_1=\frac{4}{\pi}J_c\left(E/E_c\right)^{1/N}\int_0^{\pi/2} sin(\phi)^{1+1/N}\,d\phi \tag{2}$$

with $\int_0^{\pi/2} sin(\phi)^{1+1/N}\,d\phi=\frac{\sqrt{\pi}\,\Gamma\left(1+1/2N\right)}{2\Gamma\left(3/2+1/2N\right)}$.

where Γ is the gamma function.

Another definition effective *E-J* curve is the one which links the rms value (J_{rms}) of j(t) to *E*.

If the current density j is sinusoidal with a pulsation ω, we state that $j(t) = J\,sin(\phi)$. The Fourier series expansion of $e(t)$ gives $e(t) = E_1\,sin(\phi) + ...$, so he effective E-J curves link J to E_1 or to E_{rms}. Other definitions of the E-J curve can also be adopted [5].

A. Summary of effective resistivity formulae

From the effective *E-J* curves given above, we can either define effective resistivities or conductivities as a function of *E* or *J*.

Since a magnetic vector potential formulation is adopted in this paper, effective resistivities are defined as a function of the electric field norm. We write:

$$\rho_{eq} = F_N\left(E_c / J_c\right)\left|E / E_c\right|^{1-1/N} \tag{3}$$

where ρ_{eq} and F_N are given in Table 1.

III. Application Example

The studied domain consists on a superconducting tube surrounded by air. The tube is made from BSCCO HTS material having a vacuum magnetic permeability μ_0 and a known E-J power law with N=13 and J_c=4.6 A/mm² at liquid Nitrogen temperature (77 K). The external radius of the tube is R_{ext}=4.8 mm ant its internal radius is R_{int}=4 mm. The axial length is 80 mm. The tube cross section area is about 22 mm² which gives a critical current I_c=100 A.

Both 2D (by considering an infinite length) and 3D analyses are carried out. The problem is treated using the magnetic vector potential (A_z(x,y) in 2D) and ($\vec{A} - V$ in 3D). A sinusoidal transport current $i(t)=I_{max}sin(\omega t)$ (for a frequency of 50Hz) is applied in the axial direction. In the time-harmonic FE simulations, this current is imposed via Ampere's theorem on the boundary R_{ext} of the tube. For the 3D simulation, an additional infinite box (hollow cylinder surrounding the tube) has been added at a distance far from R_{ext} and magnetic field insulation conditions have been set on its external surface.

Table 1. Effective resistivities in use

Sinusoidal e	
Equivalent resistivity used	F_N
First harmonic of j $\rho_{eq} = \rho_{es1h}$	$\dfrac{\sqrt{\pi}\,\Gamma(3/2+1/2N)}{2\Gamma(1+1/2N)}$
RMS value of j $\rho_{eq} = \rho_{esrms}$	$\sqrt{\dfrac{\sqrt{\pi}\,\Gamma(1+1/N)}{2\Gamma(1/2+1/N)}}$
Sinusoidal j	
Equivalent resistivity used	F_N
First harmonic of e $\rho_{eq} = \rho_{js1h}$	$\left(\dfrac{2\Gamma(1+N/2)}{\sqrt{\pi}\,\Gamma(3/2+N/2)}\right)^{1/N}$
RMS value of e $\rho_{eq} = \rho_{jsrms}$	$\left(\dfrac{2\Gamma(1/2+N)}{\sqrt{\pi}\,\Gamma(1+N)}\right)^{1/2N}$

The results for loss computation are presented in Fig.1 for the 2D simulations. To show the validity of the proposed method, our results are compared to those issued from a time stepping FE simulation that uses the magnetic field as state variable [3]. The FE results obtained with the frequency-domain simulations are consistent when compared to those issued from the time-stepping computation.

Compared to the transient results, it is clear that the definitions ρ_{jsrms} and ρ_{js1h} of the effective resistivity gives the worst results in evaluating the losses. For the remaining definitions of ρ_{eq}, there is a certain advantage for the definitions ρ_{es1h} and ρ_{esrms} of the effective resistivity in evaluating the losses.

The 3D simulations have been carried out for I_{max}=60A. While the computation time in 2D takes less than 1 minute, more than 2 hours are needed to achieve the 3D calculation (with approximately 2.10^5 DOF). To compare the 2D and the 3D simulations, we consider the ac losses per unit length. For an axial length of 10 mm, we obtain 8 mW/m in 3D while the 2D calculations give 3.25mW/m. This shows the need for 3D models for short HTS samples.

In the extended paper, further 3D results will be given. The computations issued from the different definitions of the effective resistivities will also be analyzed.

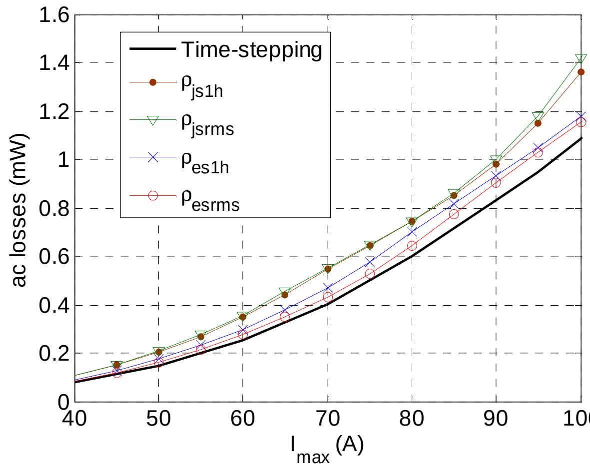

Fig. 1. Self field losses computed in 2D

References

[1] F. Grilli, S. Stavrev, Y. Le Floch, M. Costa-Bouzo, E. Vinot, I. Klutsch, G. Meunier, P. Tixador and B. Dutoit, "Finite-element method modeling of superconductors: from 2D to 3D," *IEEE Trans. Magn.*, vol. 15, No.1, pp. 17-25, 2005.

[2] A. Kameni, S. Mezani, F. Sirois, D. Netter, J. Leveque and B. Douine, "A 2D robust FE-FV mixed method to handle strong nonlinearities in superconductors," *IEEE Transactions on Magnetics*, Vol. 46, No. 8, 2010, pp. 3445 - 3448.

[3] R. Brambilla, F. Grilli and L. Martini, "Development of an edge-element model for AC losses computation of high-temperature superconductors," *Supercond. Sci. Technol.*, vol. 19, 1-9, 2006.

[4] A. G. Jack, B. C. Mecrow, "Methods for magnetically nonlinear problems involving significant hysteresis and eddy currents," *IEEE Trans. Magn.*, vol. 26, No.2, pp. 424-429, 1990.

[5] S. Mezani, B. Douine and J. Leveque, "Finite element AC-losses computation in multi-layer HTS cable using complex representation of the electromagnetic Field," *IEEE Trans. on App. Supercond.*, vol. 19, no. 3, pp. 3348 – 3351, 2009.